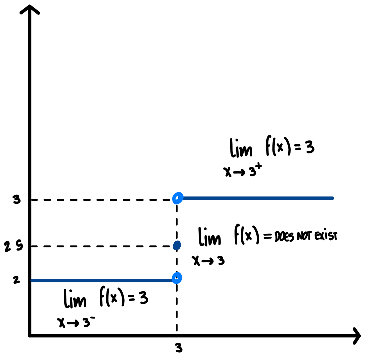
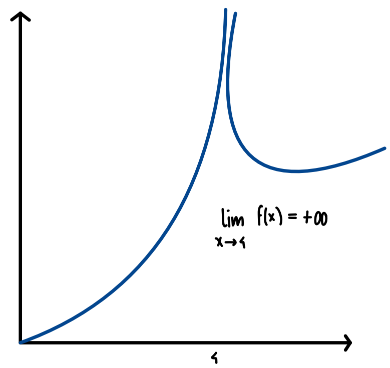
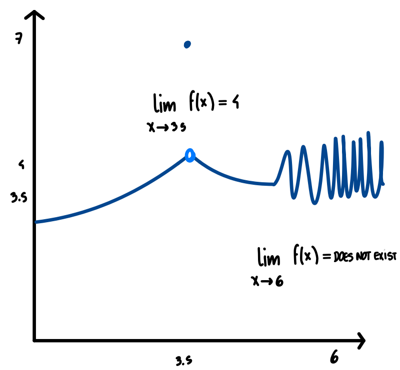
Calculus 1

MARCH 5^{th}

Limits : INTRODUCTION



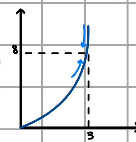




CONCEPT limits are the backbone of calculus, used, in its most basic way, to analyze the behavior of a function as it approaches a specific domain value

ex: $f(x)=x^2$

$\lim_{x\to 3} x^2 = 8$ because

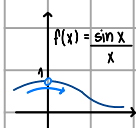


FINDING LIMITS it is possible to calculate limits by testing. Say you want the limit as x approaches 1, you can always plug in 0.999 and 1.001 or even 1 to see what the value f(x) comes close to

Now, even though, many times, limits end up, numerically, just being the output of a function, this isn't always the case and we can't just "water the term down" to that

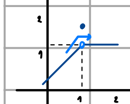
THE LIMIT IS THE VALUE A FUNCTION APPROACHES NEAR A CERTAIN INPUT, BUT NOT AT IT

let's visualize the difference through examples



NOTICE THERE IS AN INDETERMINATION AT $x=0$

However, $\lim_{x\to 0}\frac{\sin x}{x}=1$ because the function still behaves that way

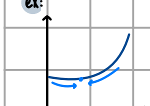


Here, even if $f(1)=2$ the limit as x approaches 1 is not 2. Think of f(x) in here as an "OUTLIER" to the function's behavior

ONE SIDED LIMITS

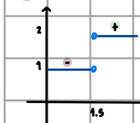
- a two-sided limit happens when f(x) approaches the same value whether it's getting closer to x from the right or left side

ex:



- a one sided limit however, happens when f(x) approaches different values from each side

ex:



$\lim_{x\to 1.5^+} f(x)=2$

$\lim_{x\to 1.5^-} f(x)=1$

LIMITS THAT DON'T EXIST

if a f(x) isn't getting closer to any real number as $x\to a$ the limit DOES NOT EXIST

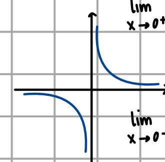
ex:

- when the one-sided limits of a function aren't equal, the TWO SIDED limit DOES NOT EXIST
- when limits increase or decrease infinitely
- when functions oscillate too much

INFINITE LIMITS AND VERTICAL ASYMPTOTES

- happens when a limit doesn't exist because it is increasing or decreasing without bound

$\lim_{x\to 0^+} f(x)=+\infty$

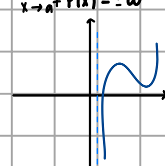


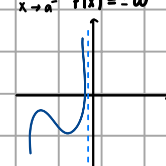
$\lim_{x\to 0^-} f(x)=-\infty$

ASYMPTOTES are when a graph rises or falls indefinitely, squeezing infinitely closer to a vertical line (THE ASYMPTOTE)

THEY HAPPEN WHEN:

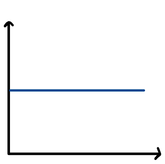
$\lim_{x\to a^+} f(x)=\pm\infty$ $\qquad$ $\lim_{x\to a^-} f(x)=\pm\infty$

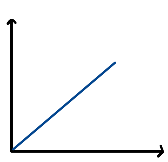


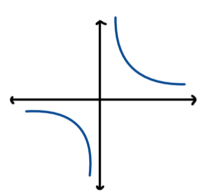


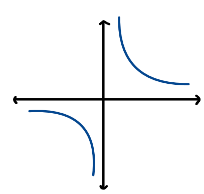
BASIC LIMITS

$\lim_{x\to a} k = k$ $\qquad$ $\lim_{x\to a} x = a$ $\qquad$ $\lim_{x\to 0^-}\frac{1}{x}=-\infty$ $\qquad$ $\lim_{x\to 0^+}\frac{1}{x}=+\infty$









Calculus 1

MARCH 9th

limits : COMPUTING

ex

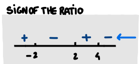
$\lim_{x\to 5} (x^2 - 4x + 3) = 5^2 - 4.5 + 3 = 8$

• In general, the limit of a polynomial $x \to a = p(a)$

$\lim_{x\to 2} \frac{5x^3+4}{x-3} = \frac{\lim_{x\to 2} 5.2^3+4}{\lim_{x\to 2} 2-3} = -44$

$\lim_{x\to 4^+} \frac{2-x}{(x-4)(x+2)} = \frac{-2}{0} = -\infty$

SIGN OF THE RATIO



$\lim_{x\to 3} \frac{x^2-6x+9}{x-3} = \frac{(x-3)^2}{x-3} = \lim_{x\to 3} x-3 = 0$

• Please review FACTORING

$\lim_{x\to 5} \frac{x^2-3x-10}{x^2-10x+25} = \frac{(x-5)(x+2)}{(x-5)^2} = \frac{x+2}{x-5} = \text{D.N.E}$

$+ \; - \; +$ (−2, 5) $\quad \lim_{x\to 5^+} f(x) = +\infty \quad \lim_{x\to 5^-} f(x) = -\infty$

PROPERTIES

1. $\lim_{x\to a} f(x) \pm g(x) = \lim_{x\to a} f(x) \pm \lim_{x\to a} g(x)$
2. $\lim_{x\to a} f(x) \cdot g(x) = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
3. $\lim_{x\to a} f(x) : g(x) = \lim_{x\to a} f(x) : \lim_{x\to a} g(x)$

• For quotient limits this rule only applies if $\lim_{x\to a} g(x) \neq 0$

if it is 0 then there are two possibilities

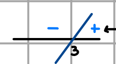
$k/0$ when $k \neq 0 \Rightarrow$ DOES NOT EXIST $(\pm\infty)$

$0/0 \Rightarrow$ FACTORING

4. $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$

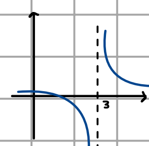
ex $f(x) = x/x-3$

$\lim_{x\to -3^+} \frac{x}{x-3} = \frac{3}{0} = +\infty$



$\lim_{x\to 3^-} f(x) = -\infty$

$\lim_{x\to 3} f(x) = \text{D.N.E}$

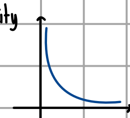


LIMITS AT INFINITY

end behavior of a function

• if you want to analyze a graph as the variable increases or decreases without bound, we use limits that approach infinity

$\lim_{x\to +\infty} \frac{1}{x} = 0$



NOTICE HOW $y=0$ BECOMES A HORIZONTAL ASYMPTOTE

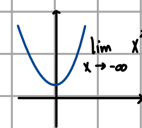
Laws and rules

1. $\lim_{x\to +\infty} (f(x))^n = \left(\lim_{x\to +\infty} f(x)\right)^n$
2. $\lim_{x\to +\infty} k.f(x) = k.\left(\lim_{x\to +\infty} f(x)\right)$

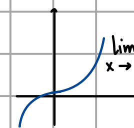
INFINITE LIMITS AT INFINITY

$\lim_{x\to +\infty} 5^x = +\infty$ because this function grows with no bond

$\lim_{x\to +\infty} x^n = +\infty$ HOWEVER $\lim_{x\to -\infty} x^n$ EVEN $= +\infty$, ODD $= -\infty$



$\lim_{x\to -\infty} x^2 = +\infty$



$\lim_{x\to -\infty} x^3 = -\infty$

Limits at infinity of polynomials

match the end behavior of the highest degree term

$\lim_{x\to -\infty} x^7 - 3x^2 + 5x = -\infty$

limits of rational functions as $x \to \pm\infty$

1. $\lim_{x\to +\infty} \frac{3x-5 \;\; \div x}{6x-8 \;\; \div x}$ DIVIDE EACH TERM BY THE HIGHEST DEGREE OF X IN THE DENOMINATOR

$\lim_{x\to +\infty} \frac{3-5/x}{6-8/x} = \frac{\lim_{x\to +\infty} 3 - \cancel{5/+\infty}^{\,0}}{\lim_{x\to +\infty} 6 - \cancel{8/+\infty}^{\,0}} = \frac{3}{6} = \frac{1}{2}$

2. METHOD 2: take the limit of the highest degree term in the numerator divided by the highest degree term in the denominator

$\lim_{x\to -\infty} \frac{8x^2-x}{2x^3-6} = \frac{8x^2}{2x^3} = \frac{4}{x} = 0$

Limits of trigonometric functions as $x \to \pm\infty$

• generally, these limits don't exist due to the constant oscilation

IN CONCLUSION

$\lim_{x\to a} k/0, k \neq 0 \Rightarrow$ SIGN ANALYSIS $\pm\infty$

$\lim_{x\to a} 0/0 \Rightarrow$ Rewrite $\to k/0$

$\to c/b$

Calculus 1

limits: epsilon-delta + CONTINUITY

EPSILON DELTA LIMIT DEFINITION

$|f(x) - L| < \varepsilon$ if $0 < |x - a| < \delta$

$\lim_{x \to a} f(x) = L$

- To prove a limit statement is true:

1. connect δ e ε
2. Prove $|f(x) - L| < \varepsilon$

ex: $\lim_{x \to 3} 4x + 5 = 17$

$|4x + 5 - 17| < \varepsilon \quad |x - 3| < \delta$

$|4x - 12| = |x - 3| < \frac{\varepsilon}{4}$

STEP 1 $\delta = \frac{\varepsilon}{4}$

STEP 2 $4|x - 3| < 4\delta = |f(x) - L| < 4\left(\frac{\varepsilon}{4}\right)$

limits at $\pm\infty$

$|f(x) - L| < \varepsilon$ if $x \lessgtr N$

infinite limits

$f(x) \gtrless M$ if $0 < |x - a| < \delta$

$\lim_{x \to 9} x - 5 = 4$

$|x - 5 - 4| < \varepsilon \Rightarrow |x - 9| < \varepsilon$

$0 < |x - 9| < \delta \Rightarrow \varepsilon = \delta$

CONTINUITY means the graph of a function has no breaks

WHAT IS CONSIDERED A BREAK:

- f is UNDEFINED at c
- Limit of $f(x)$ DNE as x approaches c
- $f(c) \neq \lim x \to c$

ex: determine which are continuous at $x = 2$

$f(x) = \frac{x^2 - 4}{x - 2} = f(2)$ is UNDEFINED $\quad \lim x \to 2 = 4$

$g(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ 3, & x = 2 \end{cases} \Rightarrow f(2) = 3 \neq \lim x \to 2 = 4$

$h(x) = \begin{cases} \frac{x^2 - 4}{x - 2}, & x \neq 2 \\ 4, & x = 2 \end{cases} \Rightarrow f(2) = \lim x \to 2 \therefore$ CONTINUOUS

- Overral, discontinuity often signals great physical change